Phenomenological Approach to Explain the Behavior of the Diffraction Peak in High-Energy Scattering*

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It is pointed out that (i) the observed shrinkage of the diffraction peaks in both p-p scattering and K^+-p scattering does not necessarily correspond to the shrinkage predicted by Regge-pole theory, (ii) an upper limit to the width of the diffraction peak can be predicted from a theoretical point of view, and (iii) the observed widths of the diffraction peaks can be explained in terms of total cross sections and a parameter having the same value for various kinds of meson-nucleon scattering (similarly for nucleon-nucleon and nucleon-antinucleon scattering).

1. INTRODUCTION

As is well known, the shrinking diffraction peaks in π-p, p-p, $\bar{p}$-p, and K-p scattering were predicted by theoretical studies based on the Regge-pole approach. On the other hand, Nambu and Sugawara[1] predicted, with some assumptions, that nonshrinking diffraction peaks should be observed at high energies, not only for π-p scattering, but also for p-p scattering. According to the experimental data,[2–4] shrinkage of the diffraction peak does not appear to be exhibited in $\pi^\pm$-p and $\bar{p}$-p scattering, whereas such shrinking is observed in p-p and K^+-p scattering. There seems to be insufficient K^--p scattering data to enable one to draw any conclusion for the shrinkage.[5] With regard to the width (Γ) of the diffraction peak, there seems to be the following experimental results[2,4]:

$$\Gamma(\bar{p}\text{-}p)\ll\Gamma(p\text{-}p)\,,\quad \Gamma(\pi^-\text{-}p)\lesssim\Gamma(\pi^+\text{-}p)\,,$$

and

$$\Gamma(K^-\text{-}p)<\Gamma(K^+\text{-}p)\,. \tag{1}$$

The purpose of this paper is to describe an attempt to explain the experimental results mentioned above.

In Sec. 2, we discuss the effects of statistics on the description of the scattering amplitude. The behavior of the p-p and K^+-p systems has been considered by many as supporting the Regge-pole theory. In Secs. 3 and 4, however, it is shown that the observed shrinkage of the diffraction peaks does not necessarily correspond to the "Regge-pole" shrinkage. The shrinking diffraction peak in p-p scattering may be explained in terms of a characteristic property of the interaction taking place in the nucleon core and of an effect due to Fermi statistics. The shrinking diffraction peak in K^+-p scattering may be explained in terms of the effect due to the energy dependence of large-angle scattering. In Sec. 5, widths of the diffraction peaks are discussed from kinematical point of view. When the diffraction peak in the small $|t|$ region (t indicates the square of the momentum transfer) is expressed in a form $|d\sigma/dt| =\exp(A_0+A_1t)$ mb/(BeV/c)^2, a lower limit for the value of A_1 can be derived by applying the unitarity condition to the S matrix. In Sec. 6, it is shown that the observed widths of diffraction peaks can be explained in terms of total cross sections, and a parameter having the same value for $\pi^\pm$-p and $K^\pm$-p scattering (similarly for p-p and $\bar{p}$-p scattering). In Sec. 7 a discussion of our empirical formula is presented.

2. SCATTERING AMPLITUDE AND STATISTICS

In order to explain the assumption which is introduced in our study, some discussion about the description of scattering is made in this section. First let us consider the scattering amplitude $f(\theta)$ for π-p or K-p scattering. In this case, $f(\theta)$ can be expressed by

$$f(\theta)=\frac{\sqrt{\pi}}{ik}\sum(2l+1)^{1/2}\left[R_l^+\left\{\frac{l+1+(\mathbf{l}\cdot\boldsymbol{\sigma})}{2l+1}\right\}+R_l^-\left\{\frac{l-(\mathbf{l}\cdot\boldsymbol{\sigma})}{2l+1}\right\}\right]Y_l^0(\theta,\varphi)\sigma(\pm\tfrac{1}{2})\,, \tag{2}$$

where $\sigma(\pm\frac{1}{2})$ indicates the spin-wave function of nucleon with $\sigma_z=\pm\frac{1}{2}$, the $[l+1+(\mathbf{l}\cdot\boldsymbol{\sigma})]/(2l+1)$ and $[l-(\mathbf{l}\cdot\boldsymbol{\sigma})]/(2l+1)$ are the projection operators for the states $J=l+\frac{1}{2}$ and $J=l-\frac{1}{2}$, respectively, and R_l^+ and R_l^- are, respectively, the R matrices for the states $J=l+\frac{1}{2}$ and $J=l-\frac{1}{2}$, with orbital angular momentum

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[4] A. N. Diddens, E. Lillethun, G. Manning, A. E. Taylor, T. G. Walker, and A. M. Wetherell, Phys. Rev. Letters **9**, 111 (1962); K. J. Foley, S. J. Lindenbaum, W. A. Love, S. Ozaki, J. J. Russell, and L. C. L. Yuan, Phys. Rev. Letters **11**, 425 (1963).
[5] If the K^--p scattering does not exhibit shrinkage of the diffraction peak, we are inclined to point out the following character with respect to the diffraction peak: In the elastic scattering of a particle A by another particle B, if there is some resonant (or bound) state in the A-B system, the diffraction peak of A-B scattering will not shrink. If there is no resonant (or bound) state in the A-B system, on the other hand, the diffraction peak of A-B scattering will shrink.

l. If the spin dependence of the R matrix is neglected, then

$$R_l^+ = R_l^- \equiv R_l = \eta_l \exp(2i\delta_l) - 1\,, \quad (3)$$

and the $f(\theta)$ reduces to

$$f(\theta) = (\pi^{1/2}/ik)\sum(2l+1)^{1/2} R_l Y_l^0(\theta,\varphi)\sigma(\pm\tfrac{1}{2})\,. \quad (4)$$

This expression is similar in form to that for the case where the spin of the nucleon is equal to zero. That is,

$$f(\theta) = (1/2ik)\sum(2l+1)R_l P_l(\cos\theta)\,. \quad (4')$$

Thus, the assumption that the spin dependence of the R matrix is neglected is equivalent in this case to the assumption that the spin of the nucleon is equal to zero.

But, it must be noted that in p-p scattering, these two assumptions lead to quite different results, as will be shown below. In the description of p-p scattering at high energies as well as at low energies, it is possible to assign the spin state of the p-p system; that is, the spin-singlet state ($S=0$) or the spin-triplet state ($S=1$), because parity (Π) can be regarded as a good quantum number in the description of strong interactions at high energy, and because the $S=0$ and $S=1$ states correspond, respectively, to the states $\Pi=+1$ and $\Pi=-1$ owing to the Fermi statistics. Thus, the well-known formula

$$d\sigma/d\Omega = \tfrac{3}{4}|f(\theta)-f(\pi-\theta)|^2 + \tfrac{1}{4}|f(\theta)+f(\pi-\theta)|^2 \quad (5)$$

is also available in the description of p-p scattering at high energy so long as the spins of the protons are not polarized. The scattering process in the case of channel spin $S=1$ can generally be described in terms of the R matrices R_l^+, R_l^0, and R_l^- for the states $J=l+1$, $J=l$, and $J=l-1$, respectively. If the assumption $R_l^+ = R_l^0 = R_l^- \equiv R_l$ is introduced, the expression for the scattering amplitude has, apart from spin-wave function, a form similar to (4′). (In the case of p-p scattering, the values of l corresponding to $S=1$ state must be odd.) From the above considerations, we can say the following: Even if the spin dependence of the R matrix is neglected,[6] both l=odd states and l=even states contribute to p-p scattering with statistical weight factors of $\frac{3}{4}$ and $\frac{1}{4}$, respectively. On the other hand, if the nucleon spin is assumed to be zero from the beginning, Fermi statistics is replaced by Bose statistics, and only the l=even states contribute to the p-p scattering.[7] Needless to say, such a rough treatment as this should not be adopted. Throughout this paper, elastic scattering at high energies is studied under the assumptions that the spin dependence of the R matrix is neglected and $\delta_l=0$.

[6] Although spin wave function was not written explicitly in the previous paper, (Ref. 10) the scattering amplitude for p-p scattering was described using this assumption.

[7] Because the first term in Eq. (5) gives, of course, a symmetric form to $d\sigma/d\Omega$, it is not correct to exclude l=odd term on the basis of $d\sigma/d\Omega$ being symmetric about 90°.

3. SHRINKING DIFFRACTION PEAK IN p-p SCATTERING

We previously adopted[8] the following expression for the elastic scattering amplitude of pion-proton scattering[9]:

$$f(\theta) = i(k/\pi^{1/2})[\exp\tfrac{1}{2}(A_0+A_1t)+C \pm\exp\tfrac{1}{2}\{B_0+B_1(u-u_0)\}](\text{mb})^{1/2}, \quad (6)$$

where

$$t=-2k^2(1-\cos\theta)\,,\quad u=(m^2-\mu^2)^2/s-2k^2(1+\cos\theta)\,,$$

s is the square of the total energy in the center-of-mass system, and u_0 is the value of u at 180°. For p-p scattering, the value of u_0 is equal to zero. Since the expression for $f(\pi-\theta)$ is given by interchanging t and u in Eq. (6), and since $\exp\frac{1}{2}(B_0+B_1u)$ or $\exp\frac{1}{2}(A_0+A_1u)$ is negligibly small in the forward scattering, $d\sigma/dt$ for elastic p-p scattering in the forward direction (0°–90°) was given by[10]

$$\begin{aligned}|d\sigma/dt| &= (\pi/k^2)[\tfrac{3}{4}|f(\theta)-f(\pi-\theta)|^2+\tfrac{1}{4}|f(\theta) + f(\pi-\theta)|^2] \cong \exp(A_0+A_1t)+\exp(B_0+B_1t)\\ &\mp\exp\tfrac{1}{2}[A_0+B_0+(A_1+B_1)t]\\ &+C[\exp\tfrac{1}{2}(A_0+A_1t)\pm\exp\tfrac{1}{2}(B_0+B_1t)]+C^2.\end{aligned} \quad (7)$$

We shall call case (I) the adoption of the upper signs of the double signs in Eqs. (6) and (7), and case (II) the adoption of the lower signs. Since $|d\sigma/dt|$ in the neighborhood of 90° is nearly equal to C^2 [cf. Eq. (7)], we can estimate the value of C by making use of the experimental data given by Cocconi *et al.*[11]

$$\begin{aligned}|C| &\cong 5\times10^{-3}\ (\text{mb})^{1/2}/(\text{BeV}/c) \quad \text{for}\ 10.8\ \text{BeV}/c\,,\\ |C| &\cong 2.4\times10^{-4}\ (\text{mb})^{1/2}/(\text{BeV}/c) \quad \text{for}\ 19.6\ \text{BeV}/c\,.\end{aligned} \quad (8)$$

Because of the smallness of $|C|$, the observed shrinkage of the diffraction peak in p-p scattering may have nothing to do with the energy dependence of $|C|$. In order to explain the shrinking diffraction peak, several attempts have been made by many authors. In their treatment, the effects due to Fermi statistics have not been taken into account in the description of p-p elastic scattering, and the shrinkage of diffraction peak has been regarded as the effect due to an energy dependence of A_1 because in this case $d\sigma/dt$ has a form similar to $[\exp\frac{1}{2}(A_0+A_1t)+C\pm\exp\frac{1}{2}(B_0+B_1u)]^2$. As another approach, we examine in this paper the effects of the second and third terms in Eq. (7) on the diffraction peak. It is shown that the experimental results for the

[8] A detailed account of our study of p-p scattering at high energies has been given previously [S. Minami, T. A. Moss, and G. A. Armoudian (to be published), and Ref. 10]. In this paper we summarize the main results and add some comments.

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[10] S. Minami, Phys. Rev. Letters 12, 200 (1964).

[11] G. Cocconi, V. T. Cocconi, A. D. Krisch, J. Orear, R. Rubinstein, D. B. Scarl, W. F. Baker, E. W. Jenkins, and A. L. Read, Phys. Rev. Letters 11, 499 (1963).

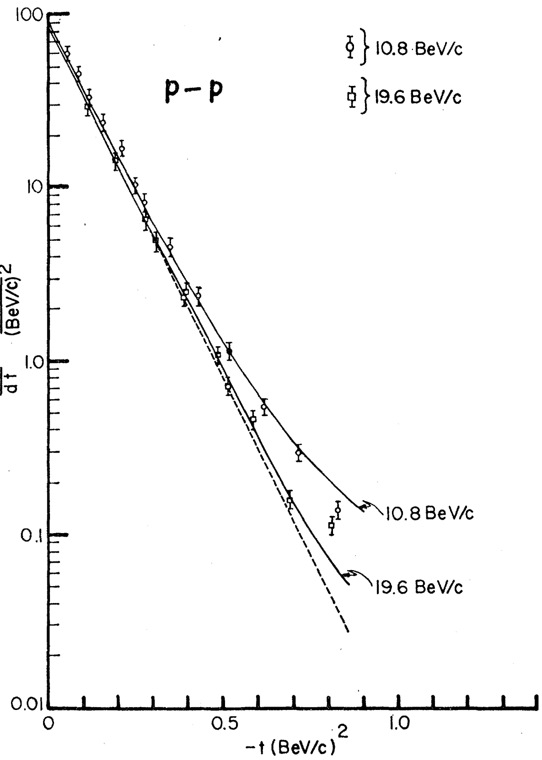

FIG. 1. Differential cross sections for elastic p-p scattering with small momentum transfer at 10.8 and 19.6 BeV/c. The experimental data are from the work of Foley *et al.* (Ref. 4). The solid curves show our results obtained by

$$|d\sigma/dt| = \exp(A_0+A_1t)+\exp(B_0+B_1t) +\exp\tfrac{1}{2}[A_0+B_0+(A_1+B_1)t] +C[\exp\tfrac{1}{2}(A_0+A_1t)-\exp\tfrac{1}{2}(B_0+B_1t)]+C^2$$

with the following values of the parameters: For incident proton momentum of 10.8 BeV/c; $A_0=4.44$, $A_1=9.34$ (BeV/c)$^{-2}$, $B_0=-1.0$, $B_1=1.74$ (BeV/c)$^{-2}$, $C=-5\times10^{-3}$. For incident proton momentum of 19.6 BeV/c; $A_0=4.44$, $A_1=9.34$ (BeV/c)$^{-2}$, $B_0=-3.50$, $B_1=1.74$ (BeV/c)$^{-2}$, $C=-2.4\times10^{-4}$. The dashed line shows the values of $|d\sigma/dt|=\exp(A_0+A_1t)$ with $A_0=4.44$ and $A_1=9.34$ (BeV/c)$^{-2}$.

shrinkage of the diffraction peak can almost be reproduced in case (II) by adjusting only the parameter B_0 with the assumption that A_0, A_1, and B_1 are energy-independent [cf. Fig. 1].

In the previous study[10] of p-p scattering at 16.7 BeV/c, expression (7) with the following values of parameters, was found to give good fits for both the small-angle and large-angle scattering: Case (I); $A_0=4.48$, $A_1=8.8$ (BeV/c)$^{-2}$, $B_0=-3.0$, $B_1=1.74$ (BeV/c)$^{-2}$, and $C=0.8\times10^{-3}$. Case (II); $A_0=4.44$, $A_1=9.34$ (BeV/c)$^{-2}$, $B_0=-2.30$, $B_1=1.74$ (BeV/c)$^{-2}$, and $C=-0.8\times10^{-3}$. Basing on these results, we now study the energy dependence of $d\sigma/dt$. In case (II), the experimental angular distribution for elastic scattering at 10.8 and 19.6 BeV/c can be well described by our empirical formula with the parameters mentioned in (9) and (10) (cf. Fig. 1).

For 10.8 BeV/c:

$$A_0=4.44,\quad A_1=9.34\ (\text{BeV}/c)^{-2},\quad B_0=-1.0,\quad B_1=1.74\ (\text{BeV}/c)^{-2},\quad C=-5\times10^{-3}. \tag{9}$$

For 19.6 BeV/c:

$$A_0=4.44,\quad A_1=9.34\ (\text{BeV}/c)^{-2},\quad B_0=-3.50,\quad B_1=1.74\ (\text{BeV}/c)^{-2},\quad C=-2.4\times10^{-4}. \tag{10}$$

In case (I), on the other hand, it is difficult to reproduce the experimental results for $d\sigma/dt$ at 10.8 BeV/c, even after adjusting not only the parameters B_0 and B_1, but also A_0 and A_1. This is mainly due to the destructive interference between $\exp\frac{1}{2}(A_0+A_1t)$ and $\exp\frac{1}{2}(B_0+B_1t)$. Thus, we can conclude that the expression (7) with lower sign [case (II)] is more promising than that with upper sign [case (I)].

As the incident proton momentum increases, the value of B_0 in case (II) decreases. In the high-energy limit, $\exp\frac{1}{2}(B_0+B_1t)$ would tend to zero, and $d\sigma/dt$ in the small $|t|$ region can be expressed approximately by

$$|d\sigma/dt|\cong\exp(A_0+A_1t). \tag{11}$$

The dashed line in Fig. 1 shows the values of $|d\sigma/dt|$ in the high-energy limit. Based on the results shown in Fig. 1, it can be predicted that the shrinking in p-p diffraction scattering will be progressively less at higher energy.

4. SHRINKING DIFFRACTION PEAK IN K^+-p SCATTERING

As was shown in Sec. 3, the estimated values of $|C|$ in p-p scattering are so small that we can discuss shrinkage of the diffraction peak without the C term, so far as p-p scattering at momenta ≥10 BeV/c is concerned. Then the following question arises: Is it possible to expect a similar situation in any other kind of scattering?

Let us consider elastic scattering involving different kinds of particles and discuss the effect of the C term on properties of the diffraction peak under the assumption that A_0 and A_1 in Eq. (6) do not depend on the incident energy. In the high-energy region, on which we focus our attention, if the values of C are very small, or if the energy dependence of C is not so remarkable, then a nonshrinking diffraction peak can be predicted. On the other hand, if the interference term $2C\exp\frac{1}{2}(A_0+A_1t)$ with positive C value decreases very rapidly as the energy increases, and if its magnitude is too large to be neglected, then a shrinking diffraction peak can be predicted. As an example of the former case, we may consider π-p scattering at 6–20 BeV/c because the values of C would be very small. This might be supported by the following experimental result for large-angle π^--p scattering at 4.95

BeV/c[12]:

$$d\sigma/d\Omega \quad \text{at} \quad 90° = 0.000_{-0.000}^{+0.002} \text{ mb/sr}.$$

As an example of the latter case, let us examine the diffraction peak in K^+-p scattering at 6.8–14.8 BeV/c. With the following values of the parameters, we try to estimate the values of $|d\sigma/dt| = \exp(A_0+A_1t)+2C \exp\frac{1}{2}(A_0+A_1t)+C^2$ in the small $|t|$ region:

$$\text{For 6.8 BeV}/c\text{:} \quad A_0=2.90, \quad A_1=5.6 \ (\text{BeV}/c)^{-2}, \quad \text{and} \quad C=0.085. \tag{12}$$

$$\text{For 14.8 BeV}/c\text{:} \quad A_0=2.90, \quad A_1=5.6 \ (\text{BeV}/c)^{-2}, \quad \text{and} \quad C=0.006. \tag{13}$$

The results shown in Fig. 2 make it possible to point out the following possibility: The shrinkage of the diffraction peak in K^+-p scattering at 6.8–14.8 BeV/c is not due to an energy dependence of A_1, but due to an energy dependence of C. Now C is regarded as one of the parameters by which the interaction in the nucleon core can be described, because it is related to an s-wave interaction. The values of C mentioned in Eqs. (12) and (13) make it possible to predict the following results:

$$|d\sigma/dt|_{\text{at } 90°} \cong 7\times10^{-3} \text{ mb}/(\text{BeV}/c)^2 \quad \text{for 6.8 BeV}/c, \tag{14}$$

and

$$|d\sigma/dt|_{\text{at } 90°} \cong 4\times10^{-5} \text{ mb}/(\text{BeV}/c)^2 \quad \text{for 14.8 BeV}/c. \tag{15}$$

On the contrary, if the energy dependence of A_1 is responsible for the shrinkage of the diffraction peak, as was suggested by Regge-pole theory, $|d\sigma/dt|$ at 90° may not have such a strong energy dependence as given above, and its magnitude for 6.8 BeV/c would be much smaller than that given by Eq. (14). Future experiments involving measurement of $d\sigma/dt$ in the neighborhood of 90° will play an essential role in deciding which interpretation should be adopted.

We think it worthwhile to add the following two remarks about the effects of C: (i) As the incident energy increases, the magnitude of C decreases, and the values of $d\sigma/dt$ tend to $\exp(A_0+A_1t)$ mb/(BeV/c)^2. The dashed line in Fig. 2 shows the values of $|d\sigma/dt| = \exp(A_0+A_1t)$. Therefore, we can predict that the shrinking in K^+-p diffraction scattering would be progressively less at high energy (cf. Fig. 2). (ii) If C has a negative sign and C^2 is a decreasing function of energy, then the diffraction peak will expand more or less as the incident energy increases. In the experimental results for $\bar{p}$-p elastic scattering at 7.2–12.0 BeV/c, there seems to be a slight tendency toward this behavior, although the data may not be sufficient to draw this conclusion.

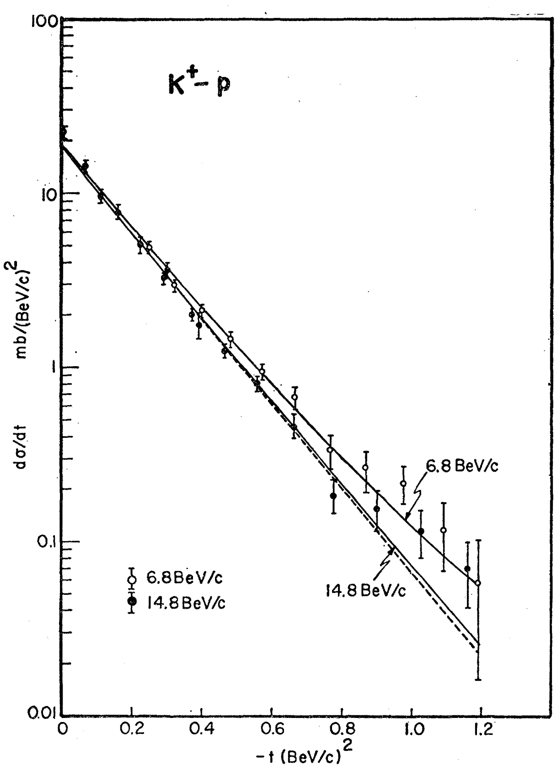

FIG. 2. Differential cross sections for elastic K^+-p scattering with small momentum transfer at 6.8 and 14.8 BeV/c. The experimental data are from the work of Foley *et al.* (Ref. 2). The solid curves show our results obtained by $|d\sigma/dt| = \exp(A_0+A_1t) +2C \exp\frac{1}{2}(A_0+A_1t)+C^2$ with the following values of parameters: For 6.8 BeV/c; $A_0=2.90$, $A_1=5.6$ (BeV/c)$^{-2}$, and $C=0.085$. For 14.8 BeV/c; $A_0=2.90$, $A_1=5.6$ (BeV/c)$^{-2}$, and $C=0.006$. The dashed line shows the values of $|d\sigma/dt| = \exp(A_0+A_1t)$ with $A_0=2.90$ and $A_1=5.6$ (BeV/c)$^{-2}$.

5. KINEMATICAL STUDY OF THE DIFFRACTION PEAK

For π-p and $\bar{p}$-p scattering processes, the diffraction peak in the small $|t|$ region can be expressed in the form $|d\sigma/dt| = \exp(A_0+A_1t)$ mb/(BeV/c)^2. As was pointed out in Secs. 3 and 4, the diffraction peaks in p-p and K^+-p scattering can also be approximately expressed by $|d\sigma/dt| = \exp(A_0+A_1t)$ mb/(BeV/c)^2 when the incident energy is sufficiently high. As is well known, the value of A_0 can be estimated from the total cross section σ_{tot}. That is[13]

$$\text{Im} f(0°) = (k/\pi^{1/2})\exp(A_0/2) = (k/4\pi)\sigma_{\text{tot}}. \tag{16}$$

Now we want to show that there is a lower limit for the value of A_1 owing to unitarity of the S matrix. Let us discuss the contribution from $\exp\frac{1}{2}(A_0+A_1t)$ to the R matrix $(=\eta_l \exp(2i\delta_l)-1)$ for the lth partial wave. When k and $\exp\frac{1}{2}(A_0+A_1t)$ are given in the units of (BeV/c) and (BeV/c)$^{-2}$, respectively,[13] the contribu-

[12] M. L. Perl, L. W. Jones, and C. C. Ting, Phys. Rev. **132**, 1252 (1963).

[13] Note that $\exp\frac{1}{2}(A_0+A_1t)$ is usually given in the unit of (mb)$^{1/2}$/(BeV/c) because $|d\sigma/dt| = \exp(A_0+A_1t)$ mb/(BeV/c)^2.

tion from the forward peak to $(1-\eta_l)$ can be expressed by

$$(1-\eta_l)'=(k^2/\pi^{1/2})\int_{-1}^{1}\exp(a_0+a_1x)P_l(x)dx\,, \quad (17)$$

where $a_0=A_0/2-A_1k^2$ and $a_1=A_1k^2$. For $l=0$ and $l=1$,

$$(1-\eta_0)'=(k^2/\pi^{1/2})[\exp(a_0)/a_1][\exp(a_1)-\exp(-a_1)], \quad (18)$$

$$(1-\eta_1)'=(k^2/\pi^{1/2})[\exp(a_0)/a_1]\{[\exp(a_1)+\exp(-a_1)]-(1/a_1)[\exp(a_1)-\exp(-a_1)]\}. \quad (19)$$

For such a high-energy scattering process that a_1 value is very large,

$$(1-\eta_0)'\cong(k^2/\pi^{1/2})[\exp(a_0+a_1)/a_1]=(1/\pi^{1/2})\exp(A_0/2)/A_1\,, \quad (20)$$

$$(1-\eta_1)'\cong(k^2/\pi^{1/2})[\exp(A_0/2)/A_1][1-1/(A_1k^2)]\lesssim(1/\pi^{1/2})\exp(A_0/2)/A_1. \quad (21)$$

Here we consider the following results which were shown in a previous paper[9]: (i) It is necessary to have detailed experimental data for large-angle scattering, particularly the data for backward scattering, in order to obtain the correct value[14] of $(1-\eta_l)$. (ii) If the backward peak exists and the contribution from the backward peak to $(1-\eta_l)$ is expressed by $(1-\eta_l)''$, the value of $(1-\eta_l)''$ associated with even l must have the opposite sign to the value of $(1-\eta_l)''$ associated with odd l. (iii) Both $(1-\eta_l)'$ and $|(1-\eta_l)''|$ are the decreasing functions of l and tend to zero as l increases. Based on these results, we can say of $(1-\eta_0)'$ and $(1-\eta_1)'$ that both or one of them, at least, must be smaller than unity owing to unitarity of the S matrix. Namely[15]

$$(1/\pi^{1/2})\exp(A_0/2)/A_1\leq 1. \quad (22)$$

The lower limits of A_1 for various kinds of scattering are shown in Table I, where the A_0 values are estimated by making use of the experimental data[2,4] for σ_{tot} [cf. Eq. (16)].

TABLE I. Widths of the diffraction peaks in high-energy scattering.

Scattering	A_0	Lower limit of A_1 (BeV/c)$^{-2}$	$(1-\eta_0)'$	A_1 (BeV/c)$^{-2}$	$\sigma_{\text{el}}\cong\exp(A_0)/A_1$ (mb)
p-p	4.44	8.34	0.893	9.34	9.08
$\bar{p}$-p	5.11	11.69	0.893	13.09	12.73
π^--p	3.56	5.37	0.69	7.79	4.52
π^+-p	3.44	5.07	0.69	7.34	4.26
K^--p	3.44	5.07	0.69	7.34	4.26
K^+-p	2.90	3.86	0.69	5.60	3.24

[14] Therefore $(1-\eta_l)$ is not generally equal to $(1-\eta_l)'$.

[15] Note that both $\exp(A_0/2)$ and A_1 are given in the units of (BeV/c)$^{-2}$.

6. WIDTHS OF THE DIFFRACTION PEAKS IN ELASTIC SCATTERING AT HIGH ENERGIES

In the optical model, the following assumption has been used in order to explain the diffraction peak.

$$1-\eta_l=\alpha \quad \text{for} \quad 0\leq l\leq L\,,$$
$$1-\eta_l=0 \quad \text{for} \quad l>L\,, \quad (23)$$

where $L=kR$, and R is the radius of the proton in this simple model. Using a similar viewpoint, we now try to adopt $(1-\eta_0)'$ as a parameter. Since σ_{tot} in the high-energy region seems to be independent of energy, A_0 can be regarded as an energy-independent parameter. Although shrinkage of the diffraction peaks has been observed in p-p and K^+-p scattering, this does not necessarily imply an energy dependence of A_1, as was shown in Secs. 3 and 4. In this case $(1-\eta_0)'$ would be an energy-independent parameter. If the diffraction peaks for various kinds of scattering can be described in terms of a single value of $(1-\eta_0)'$, then $(1-\eta_0)'$ can be regarded as a good parameter. But, as is shown below, it is necessary to consider at least two different values of the parameter $(1-\eta_0)'$, each value corresponding to a particular type of scattering. In Secs. 3 and 4 it was shown that empirical formulas with the following values of A_0 and A_1 were found to give good fits for diffraction scattering:

$$A_0=4.44\,,\quad A_1=9.34\ (\text{BeV}/c)^{-2} \quad \text{for } p\text{-}p \text{ scattering at 10–20 (BeV}/c)\,, \quad (24)$$

$$A_0=2.90\,,\quad A_1=5.60\ (\text{BeV}/c)^{-2} \quad \text{for } K^+\text{-}p \text{ scattering at 7–15 (BeV}/c). \quad (25)$$

From Eqs. (20), (24), and (25), it follows that

$$(1-\eta_0)'\cong 0.893 \quad \text{for} \quad p\text{-}p \text{ scattering}\,, \quad (26)$$

$$(1-\eta_0)'\cong 0.69 \quad \text{for} \quad K^+\text{-}p \text{ scattering}. \quad (27)$$

Based on these results, we classify the various kinds of diffraction scattering into the following two types: (i) meson-nucleon scattering (in general meson-baryon scattering), (ii) nucleon-nucleon and nucleon-antinucleon scattering (in general baryon-baryon and baryon-antibaryon scattering), and assume that values of $(1-\eta_0)'$ for the diffraction scattering belonging to (i) and (ii) are nearly equal to 0.69 and 0.893, respectively. This assumption leads to the following conclusion with respect to width (Γ) of the diffraction peak.

$$\Gamma(K^-\text{-}p)<\Gamma(K^+\text{-}p)\,,\quad \Gamma(\pi^-\text{-}p)\lesssim\Gamma(\pi^+\text{-}p)\,,$$

and

$$\Gamma(\bar{p}\text{-}p)\ll\Gamma(p\text{-}p),$$

because $\sigma_{\text{tot}}(K^-\text{-}p)>\sigma_{\text{tot}}(K^+\text{-}p)$, $\sigma_{\text{tot}}(\pi^-\text{-}p)\gtrsim\sigma_{\text{tot}}(\pi^+\text{-}p)$ and $\sigma_{\text{tot}}(\bar{p}\text{-}p)\gg\sigma_{\text{tot}}(p\text{-}p)$. In the fifth column of Table I are shown the A_1 values obtained using Eq. (20). Since the detailed explanation for the diffraction peaks for p-p and K^+-p scattering have been given in Secs. 3 and 4, we now show in Figs. 3 and 4 only the results cal-

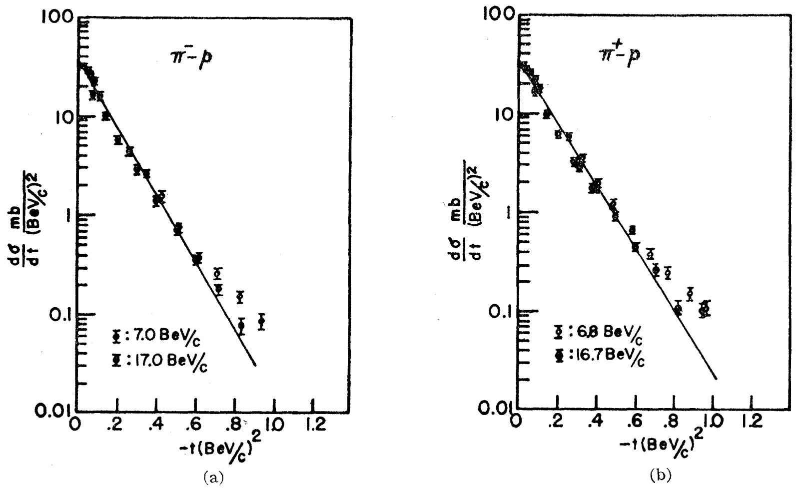

FIG. 3. Differential cross sections for elastic π^--p and π^+-p scattering with small momentum transfer. The experimental data are from the work of Foley *et al.* (Ref. 4). The solid lines show our results in the case where $\sigma_{tot}=26.2$ mb, $(1-\eta_0)'=0.69$ for π^--p scattering, and $\sigma_{tot}=24.7$ mb, $(1-\eta_0)'=0.69$ for π^+-p scattering (cf. Table I).

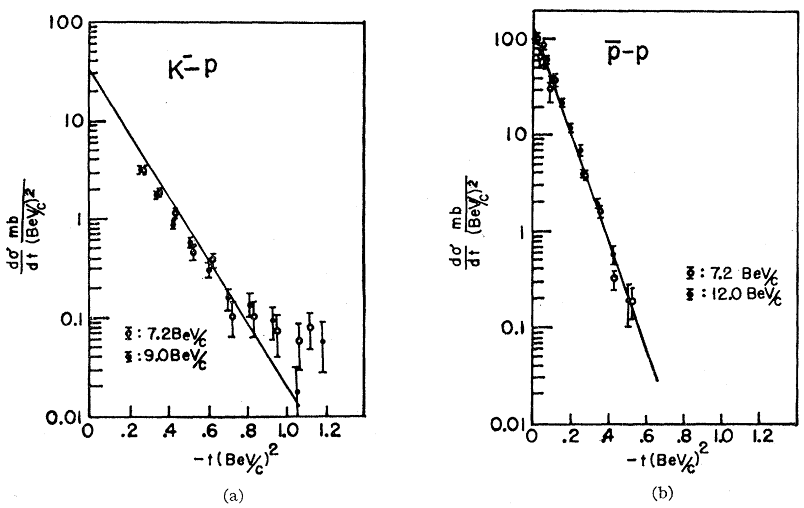

FIG. 4. Differential cross sections for elastic K^--p and $\bar{p}$-p scattering with small momentum transfer. The experimental data are from the work of Foley *et al.* (Ref. 2). The solid lines show our results in the case where $\sigma_{tot}=24.7$ mb, $(1-\eta_0)'=0.69$ for K^--p scattering, and $\sigma_{tot}=57$ mb, $(1-\eta_0)'=0.893$ for $\bar{p}$-p scattering (cf. Table I).

culated for the $\pi^{\pm}$-p, K^--p, and $\bar{p}$-p diffraction peaks. These results are in good agreement with the experimental results for the widths of these peaks. Thus, we can conclude that the widths of all the diffraction peaks in meson-nucleon scattering and nucleon-nucleon (or nucleon-antinucleon) scattering can be understood in terms of $(1-\eta_0)'$ with the values mentioned in Eqs. (27) and (26), respectively. The quantity $(1-\eta_0)'$ can be regarded as one of the good parameters in describing the diffraction scattering at high energies.

Because the total elastic cross section comes almost entirely from the diffraction peak, the elastic cross section can approximately be estimated by

$$\int (d\sigma/dt)dt \cong \int_0^{t_{\max}} \exp(A_0-A_1t)dt$$

$$\lesssim \int_0^{\infty} \exp(A_0-A_1t)dt = \exp(A_0)/A_1, \quad (28)$$

where $\exp(A_0)$ and A_1 are given in the units of mb/$(\mathrm{BeV}/c)^2$ and $(\mathrm{BeV}/c)^{-2}$, respectively. The values of $\exp(A_0)/A_1$ are illustrated in Table I. For p-p scattering, the expression $\sigma_{el}\cong\exp(A_0)/A_1$ should not be taken too seriously because one must take into account effects due to Fermi statistics.

7. DISCUSSION CONCERNING THE EMPIRICAL FORMULA

In expression (6), the first and third terms are responsible, respectively, for the forward peak and the backward peak if the latter peak exists. The parameter C has been determined so that $d\sigma/d\Omega$ at 90° may have the same value as that observed. The presence of the iC term may be interpreted as an effect due to the inelastic scattering, which might be described in terms of the statistical model. In the statistical model[16] there is a

[16] With regard to the statistical model, see the following papers: G. Fast and R. Hagedorn, Nuovo Cimento **27**, 208 (1963); G. Fast, R. Hagedorn, and L. W. Jones, Nuovo Cimento **27**, 856 (1963); L. W. Jones, K. W. Lai, M. L. Perl, S. Ting, V. Cook, B. Cork, and W. Holley, *Proceedings of the 1962 Annual International Conference on High Energy Physics at CERN*, edited by J. Prentki (CERN, Geneva, 1962) p. 591; L. W. Jones, Phys. Letters **8**, 287 (1964); G. Cocconi, CERN Report NP/GC/K1. March 5, 1964 (unpublished).

final state involving the same particles as in the initial state. The effect due to this process is not included in the iC term. This is the reason why the C term may interfere with the term $\exp\frac{1}{2}(A_0+A_1t)$.

Foley *et al.*[2,4] have adopted $d\sigma/dt=\exp(a+bt+ct^2)$ as an expression of $d\sigma/dt$ in the small $|t|$ region. This empirical formula gives better fits for the diffraction peaks than $d\sigma/dt=\exp(A_0+A_1t)$ because of the existence of an additional parameter. If we adopt, in the expression (6), $\exp\frac{1}{2}(a+bt+ct^2)$ instead of $\exp\frac{1}{2}(A_0+A_1t)$, it is difficult to obtain the empirical formula which can fit the experimental data for scattering over all angles, particularly the large angles.[17] In order to perform partial-wave analysis, it is important to take into account the character of large-angle scattering, as was emphasized previously.[9]

[17] Recently Orear tried to express $d\sigma/d\Omega$ in terms of transverse momentum $p_\perp$ [J. Orear, Phys. Rev. Letters **12**, 112 (1964)]. However, his expression is applicable in a limited $|t|$ region. Krisch has expressed $d\sigma/d\Omega$ by a sum of three exponentials [A. D. Krisch, Phys. Rev. Letters **11**, 217 (1963), and private communication].

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Measurement of the Form Factor Ratio in $K_{\mu_3}^-$ Decay

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150 000 photographs from a propane-Freon bubble chamber have been scanned for examples of in-flight $K_{\mu3}$-mode decay of 440-MeV/c K^- mesons. 150 candidates have been found from which 138 events have been selected for final analysis. These events have been analyzed assuming $K_{\mu3}$ decay to be dominated by vector coupling with form factors that may be considered energy-independent. The experimentally observed muon total-energy spectrum has been used in a likelihood calculation to determine the ratio of the form factors. The data strongly favor the ratio $\xi=0$, which yields a muon-energy spectrum favoring low-energy decay muons.

PREVIOUS investigations of form factor behavior in the three-body leptonic decays of the long-lived K mesons have utilized the decays $K^+\to\ell^++\pi^0+\nu$ and $K_2^0\to\ell^\pm+\pi^\mp+\nu(\bar\nu)$. In this paper we report a measurement of the form factor ratio in the hitherto ignored decay, $K^-\to\mu^-+\pi^0+\bar\nu$.

The usual theoretical description[1] of the generalized decay process, $K\to\mu+\pi+\nu$, assumes a universal $V-A$ interaction. For the decay $K^-\to\mu^-+\pi^0+\bar\nu$, this leads to a matrix element of the form:

$$M=\tfrac{1}{2}[f_+(q^2)Q_\lambda+f_-(q^2)q_\lambda][\bar u_\mu\gamma_\lambda(1+\gamma_5)v_\nu], \qquad (1)$$

where

$$Q_\lambda\equiv P_{K;\lambda}+P_{\pi;\lambda};\; q_\lambda\equiv P_{K;\lambda}-P_{\pi;\lambda};\; q^2\equiv q_\lambda q_\lambda, \qquad (1a)$$

and f_+ and f_-, the form factors, are scalar functions of the invariant q^2. If time-reversal invariance holds, they may be taken to be real. Their dependence on q^2 is expected to be mild and to a first approximation they may be assumed to be constants. Their ratio, $\xi\equiv f_-/f_+$, is reasonably accessible to experimental determination in several ways. The shape of the muon-energy spectrum depends on the value of ξ as does the muon longitudinal polarization. In this experiment we utilize the first approach.

In an effort to evaluate the parameter ξ from the shape of the muon-energy spectrum, we have analyzed 138 examples of the decay process: $K^-\to\mu^-+\pi^0+\bar\nu$. These events were identified in a scan of 150 000 bubble chamber photographs taken using the 30-in. Lawrence Radiation Laboratory bubble chamber. The chamber was filled with a propane-Freon mixture (24% C_3H_8–76% CF_3Br by weight) and operated in a 13-kG magnetic field. The beam particles were K^- mesons from the 800-MeV/c separated beam of Murray *et al.*,[2] degraded first to 550 MeV/c by a copper absorber upstream from the chamber, and finally to 440 MeV/c by a 1-in. copper plate placed inside the chamber 5 in. from the beam entrance. From the range distribution of stopping beam tracks, we estimate the beam momentum to be 440 ± 25 MeV/c at the downstream edge of the copper plate.

At the scanning level we have accepted 419 examples of beam particle decay satisfying the following three criteria: (1) the decay has only one charged secondary; (2) the charged secondary comes to rest in the chamber,

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[1] See, for example, J. Bernstein and S. Weinberg, Phys. Rev. Letters **5**, 481 (1960); N. Brene, L. Egardt, and B. Qvist, Nucl. Phys. **22**, 553 (1961). The relation $R\equiv W(K_{\mu3})/W(K_{e3})=0.651+0.126\xi+0.0189\xi^2$ is taken from this reference. See also P. Dennery and H. Primakoff, Phys. Rev. **131**, 1334 (1963).

[2] P. Bastien, O. Dahl, J. Murray, M. Watson, R. G. Ammar, and P. Schlein, in *Proceedings of an International Conference on Instrumentation for High Energy Physics, Lawrence Radiation Laboratory, University of California at Berkeley, 1960* (Interscience Publishers, Inc., New York, 1961).